# Goal-oriented Asteroid Mapping under Uncertainties using Sequential Convex Programming

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Operations in proximity of minor bodies demands high levels of autonomy to achieve cost-effective safe and reliable solutions. Autonomous path planning capability plays a pivotal role in this, allowing safe operations in very challenging environment. A goal-oriented proximity path planning methodology is presented here based on repeated trajectory refinement via sequential convex programming. The objective of the proposed algorithm is to refine an initial guess trajectory to best observe a set of features on the target surface with specific observation requirements while respecting physical and operational constraints. Maneuvering epochs are imposed on a given time horizon to satisfy ground-station requirements and their magnitude is limited by control capability. Navigation and control uncertainties are taken into account to obtain a robust solution within a modular and flexible implementation framework. In particular, navigation performances are assumed to be known at the maneuvering points enabling compatibility and integrability with on-board navigation filters, while control accuracy is modelled taking into account magnitude error and firing misalignments, identifying a conservative approach to guarantee the satisfaction of mission objectives under uncertainty. The methodology is applied to the case of proximity operations about the 433 Eros asteroid, due to the large amount of data available on this system thanks to the NASA NEAR mission flight data. A characterization of the operative environment is presented, highlighting the main perturbation contributions and including some of them inside the algorithm. Results of a Monte Carlo analysis performed on the surface feature distribution on a given initial trajectory show that the solution found always improves observations. Overall, this work presents an important step forward in enabling goal-oriented autonomous guidance capability for small bodies proximity operations.

# **I. Introduction**

The recent growing interest in small solar system bodies such as asteroids and comets for scientific inspection, exploitation of resources and planetary defense reasons is pushing the development of innovative engineering solutions to better investigate these celestial bodies. Ground-based observations allow preliminary characterizations of small bodies in terms of bulk properties such as orbit, mass, shape, rotational state and surface composition. The acquisition of range-Doppler radar data in addition to Optical and Spectroscopic observation revolutionized the way we look at asteroids allowing for precise shape and rotational state reconstruction [1]. A drastic improvement in the body characterization can be obtained with in-situ observations with the use of specialized and instrumented probes. Historically, the easiest way of performing proximity observations is achieved designing a spacecraft trajectory that intersects the target on its way towards its final destination performing a far flyby. This strategy usually provide, however, low resolution images with very limited observation windows. Rendezvousing with a small body and orbiting or hovering in its proximity is a challenging engineering task because it requires precise trajectory control and accurate navigation in a low gravity highly perturbed dynamical environment. Many robotic missions successfully performed scientific operations in proximity of asteroids, comets and minor planets. Fundamental milestones have been marked in the last two decades by missions such as NEAR Shoemaker [2], Dawn [3], OSIRIS-REx[4], Hayabusa [5], Hayabusa 2

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[6], and Rosetta [7], throwing the bases of modern deep-space exploration techniques. Recent resonance to the field is doubtless provided by DART [8]: the kinetic impactor developed within the framework of the Asteroid Impact Deflection Assessment (AIDA) program, an ESA-NASA collaboration to test the deflection capability of a kinetic impactor on potentially hazardous Near Earth Objects (NEO). DART targeted and successfully impacted in September 2022 the secondary asteroid of the Didymos-Dimorphos binary system. By looking at the history of asteroid exploration a pattern in the operation can be easily identified, a preliminary characterization of the targets is generally performed from a safe distance, following orbits or hyperbolic arcs accurately designed on ground. Knowledge of the system and its dynamical environment are initially provided only via ground observations. The initial proximity phase helps refining this preliminary knowledge paying the way to closer approaches. This tasks have always been performed by large instrumented probes with heavily margined trajectory control capability and limited autonomy on-board. Nowadays the Space exploration field is withstanding a transition towards the use of CubeSats, and miniaturized platforms in general, for the systematic exploration of the Solar System [9]. Their use aims at performing riskier tasks and operate in multi-agent scenarios while cooping with limited resources but at the same time demands higher levels of autonomy on-board to achieve cost-effective safe and reliable solutions, particularly for what concern proximity guidance and navigation. Current approaches for autonomous proximity operations often relies on tracking a reference trajectory previously designed and optimized on-ground. This kind of approach allows for no replanning capability, struggling in facing unforeseen events. Moreover, the design is often targeting the best solution in terms of fuel consumption or time of flight imposing weak requirements in terms of trajectory envelope that do not generally consider the body rotation and shape. The satisfaction of observation and mapping requirements, usually verified a posteriori, typically relies on the fact that the target rotation period is significantly shorter than the orbital one, leading to an higher probability of achieving global mapping. A key improvement in this direction is provided by the multi-satellite mission concept called Autonomous Nanosatellite Swarming (ANS) and related Simultaneous Navigation and Characterization (SNAC) algorithms [10]. These endow an ensemble of nanosatellites with the capability to autonomously navigate around and characterize a small celestial object with minimal ground intervention by estimating shape and gravity concurrently, and with higher performances than traditional methods [11]. However, this framework still breaks down for different reasons when more complex objectives are considered such as for example the observation of a specific set of surface features at a given ground sampling distance (GSD) or under specific illumination conditions. When the perturbing effect of solar radiation pressure and distributed gravity is considered, the very existence of stable closed orbits is compromised often leading to high station keeping costs. At the same time, replanning capability and target mapping requires accurate knowledge on the system dynamics, and precise estimation of the spacecraft state which are strongly affected by dynamics, navigation and control uncertainties. Finally, the minimization of fuel consumption, i.e. the spacecraft  $\Delta V$ budget, may not be of primary importance when performing small bodies proximity operations because of the high connectivity of the configuration space that allows for relatively fast and cheap reconfiguration. An innovative concept developed in recent years is proposing a paradigm shift towards autonomous goal-oriented approaches. The idea is that the probe is provided with the high-level objectives of the mission and the trajectory is computed autonomously on-board within a continuous replanning framework to best achieve the assigned tasks. In this field it is worth mentioning the previous work done by Surovik [12], where a sample based abstract reachability analysis performed in the control domain is proposed as a way of planning impulsive manoeuvres within a receding horizon model predictive control framework. The same concept is extended by Capolupo [13] to the case of global mapping. A similar methodology is also exploited in Earth's orbit for the development of Simultaneous Localization And Mapping (SLAM) techniques in proximity of artificial objects [14]. The author extended this methodology to the problem of global mapping and features observation in the proximity of a binary asteroid, investigating different metrics for tuning the algorithm depending on mission requirements [15]. While being very flexible to different mission scenarios and observation requirements, this approach presents a few limitations mainly due to the computational cost of computing the reachable set. In fact, despite exploiting an heuristic refinement technique this approach still requires massive trajectories propagation and potentially specialized hardware to be performed on-board. Moreover, the optimization is performed arc by arc with limited considerations on the global optimality of the solution found. An alternative or complementary methodology is proposed in this work to perform goal-oriented path planning within a Sequential Convex Programming (SCP) framework. The objective of the proposed algorithm is to refine an initial trajectory, potentially obtained with a simplified dynamical and observation model of the system, to best observe a set of features on the target surface with specific observation requirements while respecting physical and operational constraints. SCP is a direct optimization method used to solve non-convex optimization problem via sequential convexification that approximate the cost function and the constraints of the original problem in the neighbourhood of the previous step solution [16, 17]. This approach has been used in concept studies for spacecraft proximity operations [18], fuel-optimal interplanetary transfers [19], passively safe satellite swarm control [20], asteroid landing [21] and hopping [22]. To the authors knowledge the application of this methodology for the optimization of asteroids goal-oriented proximity operations has not been explored yet. The structure of this paper is reported below. In Sec.II the problem statement is described and the optical guidance problem is formulated in its non-linear continuous form. Sec.III illustrates the methodology used to map the original problem into a series of reduced convex optimization problems taking into account navigation and control uncertainties. Sec.IV illustrates the application of the proposed methodology to the case of Eros proximity operations and shows the results while, Sec.V summarizes the work contribution and states the prospective for future investigation.

# **II.** Problem statement

The methodology presented in this paper aims at solving the following problem. Given: 1) a set of features on the asteroid surface; 2) a set of observation requirements for each feature; 3) an initial spacecraft trajectory and control profile and, 4) a set of operational and safety constraints; find a trajectory that improves the performances of the original one in terms of observation opportunities under navigation and control uncertainties.

### A. From observation requirements to observation regions

Let's assume that the observation requirements for a given feature on the target surface are expressed in terms of required ground sampling distance  $GSD_{req}$  from nadir observation with an accuracy of  $\sigma_{GSD}$  one sigma, and maximum off-nadir pointing allowed  $\theta_{max}$ . By looking at Figure 1a, it is easy to show that the former, being the distance on the ground covered by a single camera pixel, can be translated into an altitude requirement of the form  $h \in [h_{min}, h_{max}]$  as

$$h_{min,max} = \frac{\left(GSD_{req} \mp 3\sigma_{GSD}\right)}{tan(iFoV)} \stackrel{iFoV \ll 1}{\sim} \frac{\left(GSD_{req} \mp 3\sigma_{GSD}\right)}{iFoV} \tag{1}$$

where *iFoV* (in rad) is the instantaneous field of view (iFoV) of the camera, meaning the FoV of the single pixel. This value is obtained simply by dividing the FoV of the instrument by its resolution. The above constraints define a spherical sector in space which can be approximated by a convex polyhedron, see Figure 1b. This convex set can always be expressed as a function of the spacecraft position in the asteroid's fixed frame  $\mathbf{r}_B$  as the region  $\Omega_i$  defined by

$$\Omega_i := \{ \mathbf{r}_B \in \mathbb{R}^3 \mid \mathbf{K}_i \mathbf{r}_B \le \boldsymbol{\gamma}_i \}$$
<sup>(2)</sup>

with  $\mathbf{K}_i \in \mathbb{R}^{n_y x^3}$ ,  $\boldsymbol{\gamma}_i \in \mathbb{R}^{n_y x^1}$ , and  $n_y$  is the number of constraints used to define the set \*. In the context of this paper  $\Omega_i$  will always be referred to as the observation region associated with the *i*-th surface feature.



Fig. 1 Geometric relation between scientific requirements (left), and operational requirements (right).

### **B.** Cost function selection

Since the goal is to maximize the observation performances of the trajectory, a good candidate figure to be optimized must be related with the time spent by the spacecraft within the observation regions. This leads, however, to a value

<sup>\*</sup>Note that even if the example reported here approximates the spherical sector with a cone section, it is always possible to discretize further the observation region and express it in the form of Eq.2

function that is discontinuous in the control domain making the problem harder to track within an optimal control framework. A natural continuous and differentiable extension of this metric can be found in the distance of the trajectory from the set  $\Omega_i$ . The distance  $d_{\Omega_i}$  of a point **s** from a set  $\Omega_i$  in a given space is defined as

$$d_{\Omega_i} := ||\mathbf{s} - \mathbf{s}_{P,\Omega_i}|| \tag{3}$$

where  $\mathbf{s}_{P,\Omega_i}$  is the projection of the point on the set. Since  $\Omega_i$  is convex, the computation of this quantity requires finding the solution to a quadratic programming optimization problem [23]. However, an analytical solution can be found if the set is expressed in the proper space by performing a change of variables. In fact, it is always possible to define the transformation  $\mathbf{y}_i = \mathbf{K}_i \mathbf{r}_B - \gamma_i$  such that Eq.2 becomes

$$\Omega_i = \{ \mathbf{y}_i \in Y \subset \mathbb{R}^{n_y} \mid \mathbf{y}_i \le \mathbf{0} \}$$
(4)

The projection  $\mathbf{y}_{P,\Omega_i}$  of a vector  $\mathbf{y}_i \in Y$  on  $\Omega_i$  is therefore given by

$$\left(\mathbf{y}_{P,\Omega_{i}}\right)_{j} = \begin{cases} 0 & \text{if } (\mathbf{y}_{i})_{j} \ge 0\\ (\mathbf{y}_{i})_{j} & \text{otherwise} \end{cases}$$
(5)

where the subscript *j* refers to the vector component. The distance from  $\Omega_i$  is then given by  $d_{\Omega_i} = ||(\mathbf{y}_i - \mathbf{y}_{P,\Omega_i})||$  with:

$$(\mathbf{y}_i - \mathbf{y}_{P,\Omega_i})_j = \begin{cases} (\mathbf{y}_i)_j & \text{if } (\mathbf{y}_i)_j \ge 0\\ 0 & \text{otherwise} \end{cases}$$
(6)

This function is continuous but non differentiable on the boundary of the observation regions, to solve this issue and induce more regularity, its square is considered and reported in Eq.7.

$$d_{\Omega_i}^2 = (\mathbf{y}_i - \mathbf{y}_{P,\Omega_i})^T (\mathbf{y}_i - \mathbf{y}_{P,\Omega_i}) = \sum_{j=1}^{n_y} max^2(0, (\mathbf{y}_i)_j)$$
(7)

Observe that,  $d_{\Omega_i}^2$  is a convex function in  $\mathbf{y}_i$ , and hence in  $\mathbf{r}_B$ , being  $\mathbf{y}_i$  an affine transformation of the latter one. The objective of the optimization is then to find the trajectory that has the closest distance from all the  $n_{\Omega_i}$  observation regions specified. This can be achieved through the minimization of the cost function specified by

$$J(\mathbf{r}_B) = \frac{1}{R_s^2} \left( \sum_{i=1}^{n_{\Omega_i}} \lambda_i^* d_{\Omega_i}^2 \left( \mathbf{r}_B \left( t_i^* \right) \right) \right)$$
(8)

where:  $R_s$  is a scale factor,  $\lambda_i^*$  are weighting coefficients that are used to improve the convergence of the algorithm as discussed in Sec.III.B and,  $t_i^*$  is the epoch in which the trajectory is closest to the observation region  $\Omega_i$ , i.e. the time in which the spacecraft is likely to perform the scientific observations. Note that this cost function is implicit since  $t_i^*(\mathbf{r}_B, \Omega_i)$ , this problem will be also assessed in Sec.III.B. The reason the cost function is evaluated only at the closest points  $t_i^*$  needs a deeper explanation. If the problem would be specialized at the case with only one observation region, i.e.  $n_{\Omega_i} = 1$ , it would be possible to evaluate the distance  $d_{\Omega_i}$  on all the trajectory. However, when dealing with multiple features, the distance from a set of disjoint observation regions is not uniquely defined. In these terms, Eq.8 provides a generalization of the distance in Eq.7 to the more general case of  $n_{\Omega_i} > 1$ .

### **C.** Constraints

Four constraints are considered in this work: 1) the differential equations constraint on the dynamics, 2) the admissible region constraint on the spacecraft position, 3) the control constraint due to actuators limitations and maneuvering schedule and, 4) an operational constraint on scientific acquisitions. Under the assumption of autonomous control-affine non-linear dynamics, the differential equations constraint of the problem can be expressed as in

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{B}(\mathbf{x})\mathbf{a}(t) \tag{9}$$

where  $\mathbf{x}$  is a generic spacecraft state representation in a given reference frame,  $\mathbf{f}$  is the dynamical drift,  $\mathbf{B}$  is the control allocation matrix and  $\mathbf{a}$  is the control acceleration induced on the spacecraft. Assuming to operate within an impulsive

maneuvering framework with  $n_m$  control actions taken at times  $t_l$ , Eq.9 can be written as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{B}(\mathbf{x}) \sum_{l=1}^{n_m} \delta(t - t_l) \mathbf{u}_l$$
(10)

with  $\delta$  being the Dirac function and  $\mathbf{u}_l$  the  $\Delta V$  executed at the epoch  $t_l$ . The admissible region constraint consists in imposing that the spacecraft position in asteroid fixed frame  $\mathbf{r}_B \in A$  within a given time horizon  $t_h$ , with:

$$A := \{ \mathbf{r}_{B}(t) \in \mathbb{R}^{3} \mid \mathbf{g}\left(\mathbf{r}_{B}\left(\mathbf{x},t\right)\right) \le \mathbf{0}, \forall t \in [t_{0},t_{h}] \}$$
(11)

Where  $\mathbf{g}$  is a generic non-linear time dependent function of the state. In particular, for the case discussed here only impact and escape constraints are considered:

$$|\mathbf{r}_B|| \le R_E \tag{12}$$

$$|\mathbf{r}_B|| \ge R_I \tag{13}$$

Where  $R_E$  is a bounding sphere centred on the target that confines the envelope of the trajectory and  $R_I$  is the Brillouin sphere of the body. The admissible region is therefore defined as:

$$A := \{ \mathbf{r}_{B}(t) \in \mathbb{R}^{3} \mid ||\mathbf{r}_{B}|| \le R_{E}, ||\mathbf{r}_{B}|| \ge R_{I}, \forall t \in [t_{0}, t_{h}] \}$$
(14)

The control constraint is defined by imposing that the  $\Delta V$ s remain bounded by the actuators control capacity  $u_l^{max}$ , i.e.  $\mathbf{u}_l \in U$  with:

$$U := \{ \mathbf{u}_l \in \mathbb{R}^3 \mid ||\mathbf{u}_l|| \le u_l^{max} \}$$

$$\tag{15}$$

Finally, the last imposed constraint is an operational requirements on the scientific acquisition stating that no target observation is possible within a given time interval  $\Delta T_m$  before and after a maneuvers. This imposes a limitation on the values that  $t_i^*$  can assume, more formally:

$$t_i^* \notin T_m := \bigcup_{l=1}^{n_m} \left[ t_l - \Delta T_m, t_l + \Delta T_m \right]$$
(16)

### D. The optimal control problem

The optimal control problem is then easily obtained combining equations from Eq.8 to Eq.15, as reported in

$$\min_{\mathbf{u}(t),\mathbf{x}(t)} \frac{1}{R_s^2} \left( \sum_{i=1}^{n_{\Omega_i}} \lambda_i^* d_{\Omega_i}^2 \left( \mathbf{r}_B \left( t_i^* \right) \right) \right) \\
\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{B}(\mathbf{x}) \sum_{l=1}^{n_m} \delta \left( t - t_l \right) \mathbf{u}_l \quad \forall t \in [t_0, t_h] \\
\mathbf{x}(t_0) = \mathbf{x}_0 \\
\mathbf{r}_B \left( \mathbf{x}(t) \right) \in A \qquad \forall t \in [t_0, t_h] \\
\mathbf{u}_l \in U \\
t_i^* \notin T_m$$
(17)

The problem in Eq.17 is a deterministic, non-convex optimal control problem. Moreover, because of the implicit definition of  $t_i^*$  discussed in Sec.II.B, the cost function is highly non-linear and the existence of a solution, even in the unconstrained case, is not guaranteed.

# **III. Methodology**

To solve the problem in Eq.17 within an SCP framework the continuous non-convex optimal control problem is first translated into a discrete convex optimization problem and then embedded in the sequential convex framework. This section shows how these steps are performed and how navigation and control uncertainties are embedded in the algorithm to robustify the solution.

#### A. Convexification and discretization

An optimal control problem with linear dynamics, affine equality constraints, inequality constraints formed by convex functions, and a convex performance index is a convex optimal control problem [16]. The elements to convexify in the problem formulation are: the non-linear dynamics, the impact constraint and, the cost function. The non-linear dynamics in Eq.10 is linearized about the given reference trajectory  $\hat{\mathbf{x}}(t)$  and the given sequence of reference maneuvers  $\hat{\mathbf{u}}_l$ . The linearized dynamics can therefore be expressed as in

$$\delta \dot{\mathbf{x}} = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Big|_{\hat{\mathbf{x}}} \delta \mathbf{x} + \mathbf{B}(\hat{\mathbf{x}}) \sum_{l=1}^{n_m} \delta\left(t - t_l\right) \delta \mathbf{u}_l$$
(18)

with  $\delta \mathbf{x}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$  and  $\delta \mathbf{u}_l = \mathbf{u}_l - \hat{\mathbf{u}}_l$ . The solution of this linear differential equation is well known in literature and is given by

$$\delta \mathbf{x}(t) = \mathbf{\Phi}(t, t_0) \delta \mathbf{x}_0 + \int_{t_0}^t \mathbf{\Phi}(t, \tau) \mathbf{B}(\hat{\mathbf{x}}(\tau)) \sum_{l=1}^{n_m} \delta(\tau - t_l) \,\delta \mathbf{u}_l d\tau$$
(19)

Discretizing the solution over a temporal mesh with  $n_t$  discretization points and, applying the property of the Dirac delta function, leads to a set of equations whose general expression is given by:

$$\delta \mathbf{x}_{k+1} = \mathbf{\Phi}(t_{k+1}, t_k) \delta \mathbf{x}_k + \mathbf{\Phi}(t_{k+1}, t_k) \mathbf{B}(\hat{\mathbf{x}}(t_k)) \delta \mathbf{u}_k$$
(20)

$$\delta \mathbf{x}_{k+1} = \mathbf{\Phi}(t_{k+1}, t_k) \delta \mathbf{x}_k + \mathbf{\Phi}(t_{k+1}, t_k) \mathbf{B} \delta \mathbf{u}_k$$
(21)

where  $\Phi(t_{k+1}, t_k)$  is the State Transition Matrix (STM) of the system that maps the state error at  $t_k$  with the state error at  $t_{k+1}$ . The STM is computed with respect to the reference trajectory following the variational equations with  $\Phi(t_0, t_0) = \mathbf{I}_{3x3}$ .

$$\dot{\mathbf{\Phi}}(t,t_0) = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Big|_{\hat{\mathbf{x}}} \mathbf{\Phi}(t,t_0)$$
(22)

Worth mentioning is also the fact that, in Eq.21 the subscript to the control is changed from *l* to *k* to indicate that the same discretization is used for state and control. This means that the vector  $\delta \mathbf{u}_k = \mathbf{0}_{3x1}$  when  $t_k \neq t_l$ . The linearization in Eq.21 allows to have a convex dynamical constraint in the form of an affine combination of the optimization variables. However, in order to be consistent with the real dynamics of the system, an additional constraint needs to be added, bounding the allowable deviation from the reference trajectory. This is represented by the variable *R* which quantifies the trust region for SCP iterations

$$||\delta \mathbf{x}_k|| \le R \tag{23}$$

The second constraint to be convexified is the impact constraint in Eq.13. This condition describes a hollow sphere and therefore is not a convex set. A relaxation is performed through linearization about the reference trajectory [24, 25], and the new condition is discretized over the time horizon as shown in Eq.24.

$$\hat{\mathbf{r}}_{Bk}^T \mathbf{r}_{Bk} \ge R_I ||\hat{\mathbf{r}}_{Bk}|| \tag{24}$$

Where the hat symbol indicates quantities evaluated on the reference trajectory. The last element to convexify is the cost function; the reason why the J in Eq.8 is not convex is due to the implicit definition of  $t_i^*$ :

$$t_i^* = \arg\min_{t \notin T_m} \left( d_{\Omega_i}^2(t) \right) \tag{25}$$

which is a concave function of  $d_{\Omega_i}$  and therefore of the spacecraft state. However, under the assumption that in the neighbourhood of the reference trajectory the epoch of the closest encounter with observation region can be assumed fixed,  $t_i^*$  can be evaluated on the reference trajectory:

$$t_i^* = \arg\min_{t \notin T_m} \left( \hat{d}_{\Omega_i}^2(t) \right) \tag{26}$$

The reason why this assumption is accurate will be discussed later when presenting the SCP framework. After performing these convexifications and imposing the discretization on the time horizon, the optimal control problem in Sec.II.D can be finally reduced in a canonical form approachable by a convex solver as

$$\min_{\mathbf{u}_{k},\mathbf{x}_{k}} \frac{1}{R_{s}^{2}} \left( \sum_{i=1}^{n_{\Omega_{i}}} \lambda_{i}^{*} d_{\Omega_{i}}^{2} \left( \mathbf{r}_{B} \left( t_{i}^{*} \right) \right) \right) \\
\delta \mathbf{x}_{0} = \mathbf{0} \\
\delta \mathbf{x}_{k+1} = \mathbf{\Phi}(t_{k+1}, t_{k}) \delta \mathbf{x}_{k} + \mathbf{\Phi}(t_{k+1}, t_{k}) \mathbf{B}(\hat{\mathbf{x}}(t_{k})) \delta \mathbf{u}_{k} \quad \forall k = 0 : n_{t} - 1 \\
\hat{\mathbf{r}}_{Bk}^{T} \mathbf{r}_{Bk} \geq R_{I} || \hat{\mathbf{r}}_{Bk} || \qquad \forall k = 0 : n_{t} \\
|| \mathbf{r}_{Bk} || \leq R_{E} \qquad \forall k = 0 : n_{t} \\
|| \mathbf{u}_{k} || \leq u_{max}^{l} \qquad \forall k = l \\
\mathbf{u}_{k} = \hat{\mathbf{u}}_{k} \qquad \forall k \neq l \\
|| \delta \mathbf{x}_{k} || \leq R \qquad \forall k = 0 : n_{t}$$
(27)

To be precise, the optimization problem in Eq.27 is convex if  $\mathbf{r}_B(\mathbf{x})$  is convex. This is naturally achieved if a Cartesian state representation, position and velocity, is selected where the state can be either expressed in an inertial frame or directly in the asteroid rotating frame. If this is not the case, an additional step to linearize the kinematic relation  $\mathbf{r}_B(\mathbf{x})$  is needed before proceeding. No assumption is performed at this stage on the mesh discretization. Since the cost function is only evaluated at the  $t_i^*$  epochs and maneuvers are different from zero only at the  $t_l$  epochs, the smallest discretization needed to solve the problem is obtained by using only these as collocation points. This leads to a strong reduction in the size of the convex optimization problem with respect to having a uniform fine mesh. For example, assuming to use Cartesian coordinates to represent the state, the number of decision variables involved is given by  $N_{var} = (n_m + n_{\Omega_i}) (3+6)$ .

### **B. SCP formulation of the problem**

The solution of the convex problem in Eq.27 provides an optimal solution in the neighbourhood of the provided reference trajectory. Searching for a global minimum requires fitting this problem within a sequential convex programming framework. In this work a slight variation of the classical SCP methodology [17] is implemented to improve its convergence properties, provide more reliability under highly non-linear dynamics, and best target observation regions. Figure2 illustrates the working principle of the implemented SCP approach that is discussed in this paragraph. The SCP can be seen as an iterative optimization problem on the variables  $\{\hat{\mathbf{x}}_k, \hat{\mathbf{u}}_k, \Phi_k, R, \lambda_i^*, t_i^*\}$ .

(Step 1) The first step consists in initializing the algorithm with a reference solution  $\{\hat{\mathbf{x}}_j, \hat{\mathbf{u}}_j, \Phi_j, R\}_0$ . The letter *j* is used for subscripts to indicate that this solution is provided on a finer horizon than the one used to solve the convex optimization, in the current implementation a 5 minutes uniform time step is adopted. This reference is used to compute the distance square  $\{d_{\Omega_i,j}^2\}_0$  of the trajectory from each observation region as in Eq.7. These distances are used to compute the weighting coefficients  $\{\lambda_i^*\}_0$  for each observation region as:

$$\lambda_{i}^{*} = \begin{cases} \lambda^{max} + \frac{\lambda^{min} - \lambda^{max}}{\max_{i} \left( \min_{j} \left( d_{\Omega_{i}}^{2} \right) \right) - \min_{i} \left( \min_{j} \left( d_{\Omega_{i}}^{2} \right) \right)} \left( \min_{j} \left( d_{\Omega_{i}}^{2} \right) - \min_{i} \left( \min_{j} \left( d_{\Omega_{i}}^{2} \right) \right) \right) & \text{if } \min_{j} \left( d_{\Omega_{i}}^{2} \right) \neq 0 \\ 2\lambda^{max} & \text{otherwise} \end{cases}$$
(28)

The expression in Eq.28 simply defines a linear variation of the coefficients between  $\lambda^{min}$  and  $\lambda^{max}$  as a function of the minimum distance of the trajectory from the observation region  $\Omega_i$ . The closest the trajectory is to the region the largest is the weight increasing the chances of entering it. Moreover a jump to  $2\lambda^{max}$  is implemented when the trajectory enters the convex set, to increase the penalty of exiting from it. For this work  $\lambda^{min} = 0.1$  and  $\lambda^{max} = 1$ . The set of epochs  $\{t_i^*\}_0$  is instead computed through Eq.25 that is now an explicit expression. A value of the initial trust region is also assumed.

(Step 2) The initial reference is transformed on the reduced horizon discussed in the previous paragraph through a proper mapping  $M\left[\{(\cdot)\}_j\right] = \{(\cdot)\}_k$ . In particular, with the operator M, the authors generally referring to any transformation that maps the problem variables from the reduced horizon domain, indicated by the subscript j, to the fine horizon one, indicated by the subscript k. The solution  $\{\hat{\mathbf{x}}_k, \hat{\mathbf{u}}_k, \Phi_k, R, \lambda_i^*, t_i^*\}_0$  is used to solve the convex problem in Eq.27



Fig. 2 Workflow of the implemented SCP framework.

obtaining 1) the optimal solution  $\{\mathbf{u}_k, \mathbf{x}_k\}$ , 2) the value of the cost function *J* and, 3) the status of the solver indicating if a solution was found. In the context of this work the CVX matlab software for disciplined convex programming [26, 27] is used, since it provides a fast prototyping framework to prove the feasibility of the approach.

(Step 3) The optimal control on the full time horizon is retrieved as  $\{\mathbf{u}\}_j = M^{-1} \left[\{\mathbf{u}\}_k\}\right]$  and this is fed into the non linear dynamics integrating Eq.10 together with Eq.22 obtaining the solution  $\{\tilde{\mathbf{x}}_j, \tilde{\mathbf{\Phi}}_j\}$  on the finer mesh.

(Step 4) This steps consists in updating the reference solution and modify the algorithm trust regions. The criterion used to do this is based on the computation of a linearization error that is defined as the maximum error between the convex solver solution,  $\mathbf{x}_k$ , and the non linear solution,  $\tilde{\mathbf{x}}_j$ , mapped on the collocation domain, i.e.  $\epsilon_{lin} = \max_k \left( \left| M \left[ \tilde{\mathbf{x}}_j \right] - \mathbf{x}_k \right| \right)$ . Three parameters  $\rho_1, \rho_2, \rho_3 \in \mathbb{R}$  are defined such that  $\rho_1 < \rho_2 < \rho_3$ . If  $\epsilon_{lin} > \rho_3$  the solution is discarded and the algorithm returns to (Step 2) with the same reference trajectory and reduced trust region given by  $R_{new} = \frac{R}{\alpha}$ , with scaling factor  $\alpha > 1$ . If  $\epsilon_{lin} \le \rho_3$  the non linear solution is accepted, the parameters  $\lambda_i^*$  and  $t_i^*$  are computed respectively through Eq.28 and Eq.25 and the trust region is adjusted according to

$$R_{new} = \begin{cases} \frac{R}{\alpha} & \text{if } \rho_2 \le \epsilon_{lin} \le \rho_3 \\ R & \text{if } \rho_1 \le \epsilon_{lin} < \rho_2 \\ \beta R & \text{if } \epsilon_{lin} < \rho_1 \end{cases} \quad \text{with } \alpha, \beta > 1$$
(29)

The values of  $t_i^*$  computed are used to recompute the new mapping  $M[\cdot]$  on the reduced domain and the solution mapped is used to provide a new reference trajectory and control for the next iterations.

(Step 5) The last step consists in performing two tasks: checking the stopping criterion, and storing the best solution

achieved so far in the process  $\{\mathbf{x}_{k,opt}, \mathbf{\Phi}_{k,opt}\}\$  in terms of number of observed surface features. In this paper, the former one is defined either by a maximum number of iterations  $n_{STOP}$  reached, or by achieving the observation of all the observation regions  $\Omega_i$ . As will be shown in the results,  $\{\mathbf{x}_{k,opt}, \mathbf{\Phi}_{k,opt}\}\$  does not necessarily corresponds to the last iteration of the SCP algorithm. The reason for this is that the proposed approach is optimizing a continuous cost function based on the distance of the trajectory from the observation regions while the actual value function, the number of observed features, is intrinsically discontinuous in the state and control domain. In other words, it is possible that during some iteration of the SCP algorithm, both the cost function and the number of crossed regions decrease.

### C. Robustness to uncertainties

Operations in proximity of small bodies are characterized by large uncertainties that can be grouped in four categories: navigation, control, dynamic uncertainties and, knowledge of the target topography. The former once are associated with the accuracy of the on-board navigation strategy, control uncertainties are related with the manufacturing, mounting and working principle of the spacecraft main engine that induce errors in magnitude and direction of the provided  $\Delta V$ . The effect of maneuvers spreading is neglected in this work. Finally, dynamic uncertainties are associated with the knowledge of the dynamical environment. Solar radiation pressure and Sun third body acceleration can play an important role when the body is small like the Didymos asteroid, see Figure3a while non-spherical gravity field effects becomes predominant with larger asteroids like Eros, see Figure3b.

These effects often requires significant station keeping costs and frequent manoeuvres to follow a precise trajectory in



Fig. 3 Comparison among perturbation magnitudes in the case of Didymos and Eros system. Environment characterization for 65803 Didymos asteroid [28] (left). 433 Eros environment, with gravity coefficients retrieved from [29, 30] (right).

the target body fixed frame. Being able of including and maybe exploiting these perturbations in the path planning problem would help to reduce a lot the design burden on the spacecraft propulsion system lowering the overall mission cost and increasing reliability. The way this is implemented within the proposed SCP framework is by properly rescaling the observation regions  $\Omega_i$  by a factor  $0 < \Gamma_i < 1$  that is a function of the maximum expected position error at time  $t_i^*$ , as defined by

$$\Gamma_i = \frac{R_{\Omega_i} - 3\sigma_r(t_i^*)}{R_{\Omega_i}} \tag{30}$$

where  $R_{\Omega_i}$  is the distance between the center of the observation region and the furthest vertex of the polyhedron, and  $\sigma_r$  is the expected standard deviation of the spacecraft position at the observation epoch  $t_i^*$ . With these scaling the convex set defined in Eq.2 becomes:

$$\Omega_i := \{ \mathbf{r}_B \in \mathbb{R}^3 \mid \mathbf{K}_i^{\alpha_i} \mathbf{r}_B \le \boldsymbol{\gamma}_i^{\alpha_i} \}$$
(31)

At each iteration of the SCP, the new quantities  $\mathbf{K}_i^{\alpha_i}$  and  $\boldsymbol{\gamma}_i^{\alpha_i}$  are recomputed during (Step 4). The way  $\sigma_r(t_i^*)$  is computed may differ and no particular requirement on the linearity or convexity of the approach is needed since the step

is performed outside of the convex problem. In this work, a linear covariance propagation is used [31] for simplicity; in particular, given the spacecraft state covariance  $\mathbf{P}(t_{lm,i})$  at the last maneuvering epoch  $t_{lm,i}$  before  $t_i^*$ , the covariance at the closest point is given by

$$\mathbf{P}(t_i^*) = \mathbf{\Phi}(t_i^*, t_{lm,i}) \mathbf{P}(t_{lm,i}) \mathbf{\Phi}^T(t_i^*, t_{lm,i})$$
(32)

The covariance matrix at  $t_{lm,i}$  is computed as

$$\mathbf{P}(t_{lm,i}) = \begin{vmatrix} \mathbf{P}_{rr}^{N}(t_{lm,i}) & \mathbf{0}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{P}_{\nu\nu}^{N}(t_{lm,i}) + \mathbf{P}_{\nu\nu}^{C}(t_{lm,i}) \end{vmatrix}$$
(33)

with  $\mathbf{P}_{rr}^{N}$ ,  $\mathbf{P}_{vv}^{N}$  and  $\mathbf{P}_{vv}^{C}$  respectively the position error covariance due to navigation error, the velocity error covariance due to navigation error and the velocity error covariance due to control uncertainties. Without lost of generality the following discussion is performed in Cartesian coordinates but it is always possible to map  $\mathbf{P}(t_i^*)$  into a different state representation. In this work, the subscripts rr and vv refers to the diagonal block matrices of the tensor while rv and vr indicates extra diagonal terms.

A simplifying hypothesis is performed on the accuracy of the navigation solution by assuming known diagonal covariance, i.e.  $\mathbf{P}_{rr}^{N}(t_{lm,i}) = \sigma_{r}^{N}\mathbf{I}_{3x3}$  and  $\mathbf{P}_{vv}^{N}(t_{lm,i}) = \sigma_{v}^{N}\mathbf{I}_{3x3}$ , with constant values of  $\sigma_{r}^{N}$  and  $\sigma_{v}^{N}$  in all control points. This condition is in line with the common assumption that the navigation filter converged before performing the next manoeuver. The control uncertainty is instead modelled taking into account magnitude error and thrust misalignment. In particular, defining a thruster reference frame as the one shown in Figure 4, the single  $\Delta V$  manoeuver can be expressed as

$$\mathbf{u}_{k}^{TF} = (1 + m_{\epsilon}) ||\mathbf{u}_{k}|| \begin{vmatrix} \cos(\alpha) \\ \sin(\alpha)\cos(\theta) \\ \sin(\alpha)\sin(\theta) \end{vmatrix}$$
(34)

where  $m_{\epsilon} \sim \mathcal{N}(0, \sigma_m)$  is the magnitude error while  $\alpha \sim \mathcal{N}(0, \sigma_d)$  and  $\theta \sim \mathcal{U}(-\pi, \pi)$  define the misalignment error. It can be shown [32] that the control covariance in the thruster reference frame is given by:

$$\mathbf{P}_{vv}^{C,TF}(u_k) = \begin{bmatrix} 2N(1+P^2) - Pu_k^2 & 0 & 0\\ 0 & N(1-P^2) & 0\\ 0 & 0 & N(1-P^2) \end{bmatrix}$$
(35)

with  $N = \frac{1}{4} (1 + \sigma_m^2) u_{lm}^2$  and  $P = e^{-\sigma_d^2}$ . Plugging the three terms in Eq.33 leads to

$$\mathbf{P}(t_{lm,i}) = \begin{bmatrix} \sigma_r^N \mathbf{I}_{3x3} & \mathbf{0}_{3x3} \\ \mathbf{0}_{3x3} & \sigma_v^N \mathbf{I}_{3x3} + \mathbf{R}_{TF2N} \mathbf{P}_{vv}^{C,TF} \left( u_{lm,i} \right) \mathbf{R}_{TF2N}^T \end{bmatrix}$$
(36)

where  $\mathbf{R}_{TF2N}$  is the rotation matrix from the thruster frame to the reference frame used to describe the dynamics and  $u_{lm,i}$  is the magnitude of the nominal maneuver at  $t_{lm,i}$ . The  $\sigma_r(t_i^*)$  coefficient used in Eq.30 to rescale the observation region is then computed diagonalizing  $\mathbf{P}(t_i^*)$  and taking the square root of its maximum value. A conservative approach is also proposed in this work to perform fast and lighter computations. A similar approach was proposed in the past



Fig. 4 Sketch showing the thruster misalignment error.

by Surovik [12], but the approach is slightly revised here. The idea is to consider the worst case scenario in which the largest state uncertainty at  $t_{lm,i}$  is in the direction that leads to the maximum deviation at  $t_i^*$ . In other words, the stretching and rotation of the uncertainty ellipsoid typical of linear covariance propagation is substituted with a more conservative uniform expansion of the initial uncertainty sphere. This is achieved by performing an eigenstructure analysis on the STM leading to

$$\sigma_{r}^{max}(t_{i}^{*}) = \sqrt{\lambda^{max}(\Phi_{rr}(t^{*}, t_{lm,i})\Phi_{rr}^{T}(t^{*}, t_{lm,i}))\sigma_{r_{lm,i}}^{max,2} + \lambda^{max}(\Phi_{rv}(t^{*}, t_{lm,i})\Phi_{rv}^{T}(t^{*}, t_{lm,i}))\sigma_{v_{lm,i}}^{max,2}}$$
(37)

Where  $\lambda^{max}$  refers to the maximum eigenvalues operator and  $\sigma_{r_{lm,i}}^{max}$  and  $\sigma_{v_{lm,i}}^{max}$  are the maximum eigenvalues respectively of  $\mathbf{P}_{rr}^{N}(t_{lm,i})$  and  $\mathbf{P}_{vv}^{N}(t_{lm,i}) + \mathbf{P}_{vv}^{C}(t_{lm,i})$ . The detailed proof of this result is left in the appendix. Apart from speeding up the computation of the error covariance, this formulation would allow to solve an another important issue. By looking at Eq.30 it is clear that there is a limit to the maximum position uncertainty allowed at  $t_i^*$  given by the point in which the observation region reduces to a point. Moreover, the more  $\Omega_i$  is compressed the more difficult for the SCP algorithm to find a trajectory that crosses it. To solve this issue a minimum value of the scale factor  $\alpha_{min}$  can be specified and Eq.37 can be solved for  $\sigma_{v_{lm,i}}^{max}$  with  $\sigma_{r_{lm,i}}^{max} = \sigma_R^N$  and  $\sigma_r^{max}(t_i^*) = R_{\Omega_i} (1 - \alpha_{min}) / 3$ . Furthermore, by neglecting the firing misalignment errors the velocity uncertainty can be expressed as  $\sigma_{v_{lm,i}}^{max} = \sigma_v^N + 3\sigma_m u_{lm,i}$  from which it is possible to retrieve the maximum magnitude of the manoeuver allowable at  $t_{lm,i}$ . This information can be used directly during (**Step 4**) to update the value of  $u_l^{max}$  in the control constraints of Eq.15. However, an abrupt change on the control constraint inside the SCP framework with respect to the reference solution can lead to problem infeasibility. To avoid this, when this phenomenon is encountered the maximum control  $u_l^{max}$  is gradually decreased through the iterations.

# **IV. Results**

The NASA mission NEAR marked the beginning of the asteroid proximity operations era, performing a rendezvous of the asteroid 433 Eros on the 14th of February 2000 [2]. Eros has an elongated shape with semi-major axis of approximately 8 km and a rotation period slightly longer than 5 h. Motion in its proximity is interesting from a dynamical perspective because the body is large and heavy enough to allow for the existence of safe closed orbit solutions while being at the same time very challenging due the relevance of high order gravitational terms. This is the asteroid for which the largest amount of data is available in literature and it is also the benchmark target used by the ANS mission concept and related algorithms [10]. For this reason, the methodology is validated simulating proximity operations about Eros. The implementation and testing of the methodology is currently done in MATLAB\_R2022a<sup>†</sup>.

### A. Set up of the test case

To validate the methodology the following test case is used. The mission goal is set as the observation of 10 features on the asteroid surface. Each feature is required to be observed at an altitude  $h \in [23, 27]$ km with maximum off-nadir pointing  $\theta_{max} = 10$ deg. The spacecraft state is represented in Cartesian coordinates, i.e.  $\mathbf{x} = [\mathbf{r}, \mathbf{v}]^T$ , with  $\mathbf{r} \in \mathbb{R}^3$  and  $\mathbf{v} \in \mathbb{R}^3$  respectively the spacecraft position and velocity in the inertial frame centred in the asteroid center of mass. This representation allows to have a cost function that is convex, and an exact control-affine dynamics. Central gravity, solar radiation pressure (SRP) and J2 effect are considered in the SCP model. The expression of the Jacobians used to compute the STM are reported in the Appendix. The relevant parameters describing the system dynamics are reported, in Table 1. The initial trajectory used as reference solution to initialize the algorithm is a circular polar orbit with semi-major axis equal to 30 km. The setting to the SCP algorithm are shown in Table 2. First, the results of a single simulation are shown focusing on the relevant details, then a Monte Carlo (MC) analysis with 500 different features distribution is performed, to test the robustness of the algorithm to different initial conditions focusing on more general metrics.

### **B.** Single case

To show the effectiveness of the approach an initial feature distribution is randomly selected from a dataset of surface features. The polar reference trajectory is shown in Figure 5 both in the inertial and in the asteroid-fixed rotating frame. Although the inertial plot gives a more intuitive and simple idea of the reference orbit, looking at the body-fixed rotating frame several considerations can be performed. First of all it can be seen that in this case most of the features selected are in the polar regions of the asteroid and close to each others which suggests that the initial reference orbit may a bit a good starting guess for the SCP approach. By looking at the impact region defined in Eq.13 and shown in

<sup>&</sup>lt;sup>†</sup>https://www.mathworks.com/products/new\_products/release2022a.html

Parameter Name	Symbol	Value
Planetary constant	μ	$4.4602 \times 10^{-4} \text{ km}^3/\text{s}^2$
Cross sectional area	$A_{CS}$	1 m <sup>2</sup>
Spacecraft mass	$m_{SC}$	12 kg
SRP reflection coefficient	$C_R$	1.25
$J_2$ term	$J_2$	0.1173

 Table 1 Dynamical parameters used in the simulations.

red in Figure 5 it is clear that a significant portion of the configuration space is taken out of the operational region. This is due to the fact that the impact constraint is defined with respect to a spherical shell. Substituting  $R_I$  with the local surface radius would generalize this to impact regions of arbitrary shape. However, for the purpose of this paper, the spherical approximation is kept for simplicity. The operational constraints in Eq.16 and the maneuvering epochs are represented respectively with red dots and red circles in Figure.5. The effect of this can be clearly seen in Figure.5b for the isolated observation region on the souther hemisphere of Eros. Here, the closest distance between the trajectory and the observation region  $\Omega_i$  would fall within a forbidden time frame, being too close to a maneuver. Therefore the associated  $t_i^*$  epoch ends up being on the next passage of the spacecraft in front of  $\Omega_i$ , as indicated by the blue star. Running the SCP on this test case leads to convergence, i.e. mapping of all the features, in 10 iterations as shown in Figure6a; moreover in this case the convergence is monotonic, confirming that the initial reference trajectory is actually close to an optimal solution. Figure6b shows instead the computed optimal control for the four maneuvers during the SCP iterations represented in the Radial-Tangent-Normal frame (LVLH frame). A relevant normal component seems to be preferred to in-plane maneuvers. This is probably happening because , the mission  $\Delta V$  is only imposed as a constraint in the problem and not as part of the cost function. Normal maneuvers allow to change the orbit inclination, as shown in Figure7a, placing the spacecraft on an orbit that is not anymore frozen to the J2 perturbation effect. In fact, in this case, the recession of the ascending node is actually exploited to intersect observation regions. Finally, Figure 7b shows the plot of the final trajectory in the asteroid-fixed frame and the effect of the propagated position uncertainties on the rescaling of observation regions. As the plot shows, where the uncertainty ellipsoid (in red in the figure) is bigger, the

Symbol	Value	Ref. Eqs/ Section
$t_h$	47 h 36 m (3 orbits)	Eq.14,17,27
$n_m$	4 equally spaced on the horizon	Eq.16,18,19
R	0.5 (initial value)	Eq.23,27,29
$ ho_0$	1	Eq.29
$ ho_1$	3	Eq.29
$ ho_2$	5	Eq.29
α	3	Eq.29
eta	1.5	Eq.29
$R_I$	18 km	Eq.13,24,27
$R_E$	180 km	Eq.12,27
$u_l^{max}$	1 m/s	Eq.15,27, Sec.III.C
$\Delta T_m$	1 h	Eq.15
$\sigma_r^N$	10 m	Sec.III.C
$\sigma_v^N$	1 mm/s	Sec.III.C
$\sigma_m$	0.01	Sec.III.C
$\sigma_d$	1 deg	Sec.III.C
CVX solver	sedumi [33](or mosek [34])	-

Table 2 SCP settings.

observation region is reduced more, according with the procedure explained in Sec.III.C.



Fig. 5 Reference trajectory used to initialize the SCP shown in the inertial frame (left), and in asteroid's fixed frame with the observation regions  $\Omega_i$  in blue (right). The black line is the trajectory in the two reference frames, the blue stars indicate the  $t_i^*$  points, the red circles show the maneuvering points. while the red dots are the portion of the trajectory in which observations are forbidden.

### C. Monte Carlo analysis

The Monte Carlo analysis is performed generating a 5000 dataset of random samples on the surface of the asteroid, as shown in Figure8. Each instance of the MC select a subset of 10 different features on the surface and generates the associated observation regions. The initial reference trajectory used to initialize the SCP is still the same polar orbit discussed in Sec.IV.B. This analysis allows to test the importance of the initial guess given a set of features distribution on the surface.



Fig. 6 Cost function decrease during SCP iterations (left) and, control profile for each manoeuvre during the SCP iterations (right). The  $\Delta V$  is expressed in the Radial-Tangent- Normal (RTN) reference frame with the three components identified respectively by the red, green and blue curves.



Fig. 7 Solution of the SCP problem shown in the inertial trajectory (top) and, in the asteroid's fixed frame with the original observation regions (in blue) and the scaled observation regions in green (bottom). The red ellipsoids indicates the propagated position covariance of the spacecraft at  $t_i^*$ . To make the ellipsoids visible in the picture they have been amplified with a factor of 5.



Fig. 8 Features distribution for the Monte Carlo analysis. 500 sets of 10 features each are randomly selected on the asteroid surface, each instance of the MC consists in selecting one of these sets to generate the observation regions.

In Figure9 the statistical distribution of the obtained control profile is shown. Differently from the case described in the previous paragraph here tangential maneuvers are dominant. This could be expected in a statistical sense because of the geometry of the asteroid. In fact, since observation requirements are defined in terms of altitude from the feature, due to Eros elongated shape the observation regions are generally located at different distances from the asteroid center of mass. Therefore, in order to reach them or getting closer to them starting from a circular polar orbit, eccentricity needs to be changed. The most effective way of doing that in a circular orbit is via tangential manoeuvres. An interesting metric to consider in order to assess guidance performances is the average of the distances  $d_{\Omega_i}(\mathbf{r}_B(t_i^*))$  over all the observation regions, computed as:

$$\bar{d}_{\Omega_i} = \frac{1}{n_f} \sum_{i=1}^{n_f} d_{\Omega_i}(\mathbf{r}_B(t_i^*))$$
(38)

this quantity is clearly strongly connected to the cost function defined in Eq.8, but provides a more physical understanding. Figure 10a shows on the x-axis the initial average distance  $\bar{d}_0$ , and on the y-axis the one of the final solution found  $\bar{d}_f$  for all 500 samples of the MC. The straight line indicates the function  $\bar{d}_f = \bar{d}_0$ , therefore a point below this line indicates that the solution found is closer to set of observation regions in a mean sense. The lower the value of  $\bar{d}_f$  the better the algorithm is performing. As expected the plot shows a linear trend of the performance as the initial distance increases but with a proportionality coefficient lower than one. This means that while starting from a good guess provides better results, even the cases in which the reference trajectory is significantly far from the observation set, even up to 7 km on average, can lead to final solution much closer to the observation regions. Another important metric to observe in the sequential programming approach is the thrust region R defined in Eq.23. Figure 10b shows that the thrust region tends to increase during the SCP iterations. This suggests that the step between iteration is progressively decreasing leading to trajectories that are closer to the reference and exhibits therefore lower non-linear errors. In other words this means that the convex optimization is converging to a local minimum that is inside the convex region defined by the problem's constrains and not on its boundary, symptom that the problem is well posed.



Fig. 9 Control  $u_k$  given at the maneuvering epochs expressed in the RTN reference frame.



Fig. 10 Monte Carlo SCP performances. In blue the final mean distance of the trajectory from the observation regions as a function of the initial one (left) and, the evolution of the thrust region during the SCP iterations (right).

## V. Conclusion and future works

This work has been developed to answer a growing need for autonomy in the field of autonomous goal-oriented proximity operations to small bodies. The paper aims at answering the following question: how can a set of high-level scientific and operational requirements be translated in an optimal path-planning problem and, what techniques can be used to solve this problem in a systematic and robust framework? The state of the art in asteroids proximity guidance is limited to reference trajectory tracking or sample based techniques. These approaches often require high costs in terms of station keeping or computational resources, being also very sensible to uncertainties. This paper introduces a new approach to perform goal-oriented autonomous planning in proximity of small bodies. The methodology consists in

mapping the scientific constraints into a set of convex observation regions in the physical space. An optimal control problem is then formulated with the aim of minimizing the trajectory distance from these regions while being compliant with safety and operational constraints. This problem is solved in the framework of sequential convex programming progressively convexifying the dynamics, some operational constraints and, updating the definition of the cost function among iterations. A new way of taking into account uncertainties in the process is proposed considering navigation and control errors. In particular a conservative approach is introduced to estimate the magnitude of the position error during feature observations and bound it acting on the control given. Results seems very promising indicating that the method is statistically capable of significantly increasing the quality of a reference trajectory in terms of observation performances, even under highly non-linear and uncertain environment. The Monte Carlo analysis conducted indicates that a good guess trajectory is needed to obtain good performances; this can be either designed ad hoc for the application or obtained by a rougher reachability analysis, maybe with simplified dynamical model and without considering uncertainties. In this case, an abstract reachability analysis approach could be followed to compute a rough initial trajectory and determine the maneuvering epochs, substituting the heuristic mesh refinement part of the methodology with the solution of an SCP problem. Furthermore, new data-driven techniques could be also exploited to warm start the optimization problem. Only uncertainties in navigation and control are considered in the proposed methodology, however, the approach is easily extendible to dynamics and target uncertainties. The former ones refer to the limited knowledge of the environment or the accuracy with which this is considered inside the SCP algorithm. These can be taken into account including higher order dynamics in the on-board STM, adding a process noise to the covariance propagation, and framing the guidance algorithm within a Model Predictive Control (MPC) finite-horizon framework. Target uncertainties are instead related to the knowledge of the shape of the asteroid. The current approach is considering perfect knowledge of the target but the uncertainties in the features location could be translated into a shrinking factor for the observation regions, in the same way proposed for navigation and control uncertainties. The accuracy of linear covariance propagation should be properly assessed considering the highly non-linear dynamical environment, trading off its performance against the advantage of having closed form analytical solutions such as the one provided by Eq. 37.Fast prototyping in MALTAB suggests feasibility for on-board implementation, but processor-in-the-loop testing on flight representative hardware is needed to fully verify the implementation capability. Moreover, state representations different from the Cartesian one, could help improve the convergence performances of the SCP algorithm.

# Appendix

### A. Dynamics and Jacobian in Cartesian inertial frame

In the case of Cartesian coordinates in the inertial frame, the uncontrolled dynamics can be expressed as:

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} + C_{SRP} \frac{\rho}{\rho^3} + \mathbf{R}_{N2B}^T \mathbf{a}_B$$
(39)

Where  $C_{SRP}$  and  $\rho$  are given by Eq.40 and Eq.41.

$$C_{SRP} = \frac{P_0 * AU^2 * A_{cs} * C_R}{m_{sc}}$$
(40)

$$\boldsymbol{\rho} = \mathbf{r} - \mathbf{r}_s(t) \tag{41}$$

and  $\mathbf{a}_{\mathbf{B}}$  is the gravitational acceleration due to distributed gravity naturally expressed in body frame. The solar pressure at 1 AU is set to  $P_0 = 4.539807335646850 \times 10^{-9}$ kPa, AU = 149597870.691km and  $\mathbf{r}_s(t)$  is the asteroid sun vector as a function of time in the inertial frame.

The Jacobian of the dynamical drift  $\mathbf{f}$  with respect to the spacecraft state is needed to integrate Eq.22 and obtain the STM. In Cartesian coordinates we have:

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} \mathbf{0}_{3x3} & \mathbf{I}_{3x3} \\ \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{r}} & \mathbf{0}_{3x3} \end{bmatrix}$$
(42)

With:

$$\frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{r}} = -\mu \frac{\mathbf{I}_{3x3} r^3 - 3r \left(\mathbf{r} \mathbf{r}^T\right)}{r^6} + C_{SRP} \frac{\mathbf{I}_{3x3} \rho^3 - 3\rho \left(\rho \rho^T\right)}{\rho^6} + \mathbf{R}_{N2B}^T \frac{\partial \mathbf{a}_B}{\partial \mathbf{r}_B} \mathbf{R}_{N2B}$$
(43)

The derivative of the acceleration with respect to the position in the asteroid fixed frame can be found by differentiating the gravitational potential obtained with the classical spherical harmonic expansion [35]. This process is performed neglecting tesseral harmonics contribution and keeping only the J2 term of the zonal expansion. The obtained potential is then differentiated symbolically in Mathematica leading to the expression shown in Eq.44.

$$\frac{\partial \mathbf{a}_{B}}{\partial \mathbf{r}_{B}} = K_{J2} \begin{bmatrix} r^{2} \left(-\eta^{2} + 4r_{z}^{2}\right) + 5r_{x}^{2} \left(\eta^{2} - 6r_{z}^{2}\right) & 5r_{x}r_{y} \left(\eta^{2} - 6r_{z}^{2}\right) & 5r_{x}r_{z} \left(3\eta^{2} - 4r_{z}^{2}\right) \\ 5r_{x}r_{y} \left(\eta^{2} - 6r_{z}^{2}\right) & r^{2} \left(-\eta^{2} + 4r_{z}^{2}\right) + 5r_{y}^{2} \left(\eta^{2} - 6r_{z}^{2}\right) & 5r_{y}r_{z} \left(3\eta^{2} - 4r_{z}^{2}\right) \\ 5r_{x}r_{z} \left(3\eta^{2} - 4r_{z}^{2}\right) & 5r_{y}r_{z} \left(3\eta^{2} - 4r_{z}^{2}\right) & -3\eta^{4} + 24r_{z}^{2}\eta^{2} - 8r_{z}^{2} \end{bmatrix}$$
(44)

With:

$$K_{J2} = \frac{3J_2\mu r_0^2}{2r^9} \tag{45}$$

$$\eta = \sqrt{r_x^2 + r_y^2} \tag{46}$$

### B. On the formulation of conservative uncertainty propagation

Assuming that the initial covariance is a block diagonal matrix with null cross covariance, e.i.  $\mathbf{P}_{0rv} = \mathbf{P}_{0vr} = \mathbf{0}_{3x3}$ , the following statement holds.

$$\mathbf{P}_{rr} = \mathbf{\Phi}_{rr} \mathbf{P}_{0rr} \mathbf{\Phi}_{rr}^T + \mathbf{\Phi}_{rv} \mathbf{P}_{0vv} \mathbf{\Phi}_{rv}^T$$
(47)

A conservative estimate of the position error  $(\sigma_r^{max})^2$  is given by the spectral radius of the position covariance  $\mathbf{P}_{rr}$  given by:

$$\lambda^{max} \left( \mathbf{P}_{rr} \right) = \lambda^{max} \left( \mathbf{\Phi}_{rr} \mathbf{P}_{0rr} \mathbf{\Phi}_{rr}^{T} + \mathbf{\Phi}_{rv} \mathbf{P}_{0vv} \mathbf{\Phi}_{rv}^{T} \right) \le \lambda^{max} \left( \mathbf{\Phi}_{rr} \mathbf{\Phi}_{rr}^{T} \left( \sigma_{0rr}^{max} \right)^{2} + \mathbf{\Phi}_{rv} \mathbf{\Phi}_{rv}^{T} \left( \sigma_{0vv}^{max} \right)^{2} \right)$$
(48)

Since the tensors  $\Phi_{rr}\Phi_{rr}^{T}$  and  $\Phi_{rv}\Phi_{rv}^{T}$  are symmetric they can be always diagonalized and therefore the following inequality holds:

$$\lambda^{max} \left( \boldsymbol{\Phi}_{rr} \boldsymbol{\Phi}_{rr}^{T} \left( \boldsymbol{\sigma}_{0rr}^{max} \right)^{2} + \boldsymbol{\Phi}_{rv} \boldsymbol{\Phi}_{rv}^{T} \left( \boldsymbol{\sigma}_{0vv}^{max} \right)^{2} \right) \leq \lambda^{max} \left( \boldsymbol{\Phi}_{rr} \boldsymbol{\Phi}_{rr}^{T} \right) \left( \boldsymbol{\sigma}_{0rr}^{max} \right)^{2} + \lambda^{max} \left( \boldsymbol{\Phi}_{rv} \boldsymbol{\Phi}_{rv}^{T} \right) \left( \boldsymbol{\sigma}_{0vv}^{max} \right)^{2}$$
(49)

This leads to the expression provided in Eq.37. However, a more conservative estimate can be computed. Considering that the Cauchy-Green tensor  $\mathbf{C} = \mathbf{\Phi}\mathbf{\Phi}^T$  is positive definite by definition it means that  $\lambda^{max} \left(\mathbf{\Phi}_{rr}\mathbf{\Phi}_{rr}^T\right) \in \mathbb{R}^+$  and  $\lambda^{max} \left(\mathbf{\Phi}_{rv}\mathbf{\Phi}_{rv}^T\right) \in \mathbb{R}^+$ . Therefore, the triangular inequality holds:

$$\sigma_r^{max} \le \sqrt{\lambda^{max} \left( \Phi_{rr} \Phi_{rr}^T \right)} \sigma_{0rr}^{max} + \sqrt{\lambda^{max} \left( \Phi_{rv} \Phi_{rv}^T \right)} \sigma_{0vv}^{max}$$
(50)

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